Spring 2006

"I really *can* do math!"

Developing Mathematical Skills in Elementary Students

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How am I supposed to help students with math? I always hated math in school.

What if I don't know the answers to the problems?

I don't remember much about math. How do you divide fractions again?

I was a good student, but what if they are doing that "new" math I've heard about?

Math was always easy for me. How can I help a student who is struggling?

Volunteer tutors and mentors who help students with math homework often have concerns like these. Ask a typical group of volunteers if they consider themselves mathematically proficient, and chances are many will say no. Many adults left school with a bank of memorized rules but only a hazy understanding of mathematics, accepted because teachers, students, and parents alike believed success in mathematics was a matter of innate ability rather than effort—you either understand it or you don't.

Today, educational opportunities and jobs increasingly require a sound understanding of mathematics, leading to changes in the way mathematics is taught in school. Volunteers may be unfamiliar with the new "language" of mathematics instruction, and even those who enjoyed mathematics when they were in school may have concerns about working with students.

Greater understanding of mathematics will be essential for today's schoolchildren. Success in tomorrow's job market will require more than computational competence. It will require the ability to apply mathematical knowledge to solve problems... [Students] need to have the mathematical sophistication that will enable them to take full advantage of the information and communication technologies that permeate our homes and workplaces. Students with a poor understanding of mathematics will have fewer opportunities to pursue higher levels of education and to compete for good jobs.

—Helping Children Learn Mathematics

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- Brainstorm solutions
- Design and deliver training

In this issue of *the Tutor*, we will explore the five components of effective mathematics learning identified by the National Research Council (Kilpatrick & Swafford, 2002)—engaging, understanding, computing, applying, and reasoning—and provide definitions, practical strategies, and sample language to help you, the volunteer tutor or mentor, support students struggling with math homework. Each section will highlight strategies that can help elementary students develop mathematical proficiency. The examples primarily focus on students' learning about numbers and operations—addition, subtraction, multiplication, and division—because an understanding of numbers is the foundation for using mathematics and learning other mathematical topics.

How Has Mathematics Instruction Changed?

A shift in the definition of what it means to be mathematically proficient has led to changes in the way mathematics is taught. In the past, students in elementary school were considered successful when they had memorized procedures well enough to complete many simple problems in a short amount of time. Many people remember the timed-test of arithmetic problems—some with dread, others with fondness.

The focus on memorization is no longer adequate. Being able to think mathematically involves much more than memorizing procedures. It is the key to being able to apply mathematics to unfamiliar and complex situations.

The National Research Council (Kilpatrick & Swafford, 2002) has proposed a comprehensive definition of mathematics proficiency, made up of five components or strands. These five strands are intertwined, meaning that they are not developed in isolation but are interrelated.

- **Engaging**: Seeing mathematics as sensible, useful, and doable—*if* you are willing to work at it
- **Understanding**: Comprehending mathematical concepts, operations, and relations—knowing what mathematical symbols, diagrams, and procedures mean
- **Computing**: Carrying out mathematical procedures, such as adding, subtracting, multiplying, and dividing numbers flexibly, accurately, efficiently, and appropriately
- **Applying**: Being able to formulate problems mathematically and to devise strategies for solving them using concepts and procedures appropriately
- **Reasoning**: Using logic to explain and justify a solution to a problem or to extend from something known to something not yet known

Then and Now: What's the Difference Between Arithmetic and Mathematics?
Many volunteers likely grew up doing "arithmetic" rather than "mathematics." You might be wondering just what the difference is, anyway? We decided to "Ask Dr. Math" (see *Resources*). Here's what we found out: "Arithmetic is just one branch of mathematics, namely that involving basic techniques of calculation with numbers. When you add, subtract, multiply, divide, take square roots, and so on, you are doing arithmetic." Mathematics, on the other hand, is "the science of numbers and their operations, interrelations, combinations, generalizations, and abstractions."

Engaging: Helping students see the purpose of mathematics

Mathematically proficient people believe mathematics is useful. They see themselves as capable of learning and doing mathematics. This doesn't mean that all students will think mathematics is easy, but they will believe that they can understand what they are learning with the appropriate amount of effort.

National service members and volunteers providing tutoring and homework assistance can help their students strengthen this strand of mathematical proficiency by confronting their own and their students' math anxiety, showing students how they already use math in their daily lives, and encouraging students' persistence when working on mathematics.

Confronting math anxiety

Many adults and children commonly experience math anxiety. How many times have you heard someone say "I hate math" or "I can't do math"? You may even have said or thought these things yourself.

Math anxiety is widespread in part because negative attitudes about mathematics are usually accepted. This undermines students' ability to become mathematically proficient, so careful responses to expressions of math anxiety are essential.

When students say "I don't like math," or "I'm not good at math," it can be tempting to reassure them by responding with something like "Oh, I was never good at math, either." While you may intend this statement to comfort a struggling student, you might be reinforcing the child's belief that mathematics is a discipline only some people can learn. A better response might be "I used to think that I would never understand math, but now I know that you just have to keep working at it."

If students express frustration or negative attitudes, tutors and mentors can help them take on a different perspective. "Engaging students in mathematics is the first step toward changing negative attitudes" (Martinie, 2006). Once students are involved in solving mathematical problems, they need support and encouragement to keep going. See the sidebar above for some suggested responses to common student statements.

Confronting math anxiety does not mean you should pretend to be more confident than you are. When you don't know the answer to a problem, think of it as an opportunity to model an eagerness to find the answer and confidence in your ability to make sense of mathematics. With your student(s), use classroom texts, previous quizzes or worksheets, or Web-based resources (see *Resources* section) to track down the information you need to complete the work at hand.

Encouraging Comments

When students say:	You may be tempted to say:	A more effective response:
"I don't get it." "It's too hard."	"Math can be hard. I'll give you a hint."	"Let's see what this problem is all about. Tell me what you know so far."
"I can't."	"Just keep trying."	"Tell me about what you have done so far. What were you thinking when you tried these methods?"
"I don't know."	"Didn't you learn that in class?"	"Think about what you *do* know about this type of problem. What have you done like this before?"

Showing students how they already use math in their daily lives

Most students use math every day, whether it's figuring out how to split a slice of pizza with a friend at lunch time, comparing the standings of their favorite sports team against competitors, or calculating how much time is left until recess. The conversation below demonstrates how tutors can engage students in conversations that help them realize the ways they are already using math in their daily lives.

Tyler: *So, I know it was your birthday this weekend. Did you get any cool presents?*

Jay: *Yeah, I got some games, some new clothes, and $40 from my grandma to spend on anything I want.*

Tyler: *Did you buy anything with it yet?*

Jay: *Yeah, comic books!*

Tyler: *How many did you get?*

Jay: *I got six.*

Tyler: *How much did they cost? Did you have any money left over?*

Jay: *No, they were $7 each, so I actually had to ask my mom for $2 more.*

Tyler: *So, you used some math skills to figure out how many comic books you could buy, didn't you?*

Jay: *Yeah, I guess I did. I knew they were $7 each, so I divided 40 by 7 and I got 5 with 5 left over. That's how I knew I had to ask my mom for $2 more to get the last one, because 5 + 2 = 7.*

Conversations like these help students appreciate the value of mathematics and help volunteers learn more about the interests and hobbies of the students with whom they are working.

Encouraging persistence

Persistence, rather than innate ability, is the key to success with mathematics. Students need opportunities to see the value of persistence by solving challenging problems (Kilpatrick, Swafford, & Findell, 2001). When students become frustrated or hit an obstacle, help them break the task down into smaller pieces.

Helen is helping Robert get started with the following problem: 42 x 4

Helen: *How many 10s are in 42?*

Robert: *Four.*

Helen: *Can you write a number that represents 4 tens?*

Robert writes "40."

Helen: *What does the 2 represent in 42?*

Robert: *2 ones.*

Helen: *How could you write 42 with 10s and ones?*

Robert: *40 + 2.*

Helen: *Now, how would you multiply 4 times 40?*

Robert: *40 + 40 + 40+ 40 and that equals 160.*

Helen: *How would you multiply 4 times 2?*

Robert: *That's easy. I know that is 8.*

Helen: *Do you think you can finish the problem?*

Robert: *160 + 8 is 168. That's the answer.*

Providing recognition for students' efforts to learn helps encourage persistence. When a student has worked hard to make sense of a concept or to find a solution, make sure she recognizes her efforts. For example: "I can see that you tried several ways of solving that problem and checked your work each time until the answer made sense. Great work!"

While persistence is important, it is also possible for students to become so frustrated that they cannot make progress. If a student is struggling to learn a concept, look for some alternative ways to approach it. If you have access to a school-day teacher at your service site, he can suggest alternative approaches; if not, connect with fellow volunteers, your site supervisor, or even other students in your homework group for ideas. Repeating the same type of problem is not likely to help students move forward, especially if they become frustrated.

Tools and Tricks: What's Inside a Math Tutor's Tote Bag?

Once you begin to tutor children in math, you'll quickly realize that hands-on materials help them better understand mathematical concepts. In addition to the things you normally bring to tutoring sessions (extra pencils, paper, scissors, etc.), here are a few other tools you may want to add to your tote bag:

Tool: Cardboard
Trick: FRACTIONS—Cut a circle to represent a pizza pie (decorating it with toppings is optional) and then divide it into slices to help demonstrate fractions. Students will be able to identify the fractional parts and put them back together to see the whole.

Tool: Chocolate bar (sectioned for easy breaking)
Trick: FRACTIONS—Have the student count the number of sections on the chocolate bar before breaking any. To make fractions tangible, the student breaks one section at a time and discusses what each section represents in fractions. (If you plan to let students eat the chocolate sections, make sure students aren't allergic to chocolate or other ingredients such as peanuts!)

Tool: Dice (two)
Trick: ADDITION, SUBTRACTION, MULTIPLICATION, ORDER—Arrange dice in order from high to low or have students multiply or add dice to help make these mathematical concepts concrete and give the student more ways to practice addition and multiplication. To increase difficulty, add another die.

Tool: Index cards
Trick: PLACE VALUE—Number index cards 1–10. Deal two cards (any combination) and have the student read the number and identify the ones and 10s. Ask questions like, "How many ones are in this number, how many 10s?" As the student becomes more proficient, increase the number of cards you lay down.

Tool: Money (coins)
Trick: ADDITION, SUBTRACTION—Besides teaching basic monetary denominations, coins are especially helpful for demonstrating practical applications of mathematical concepts like addition and subtraction.

Tool: Everyday objects for counting (buttons, bottle caps, shells, etc.)
Trick: ADDITION, SUBTRACTION—Manipulatives can be as simple as a box of paperclips to help students with addition and subtraction.

Tool: Grid paper
Trick: ANY—Grid paper is especially helpful for students who have difficulty adding, subtracting, doing long division or multiplication because they can line up rows of numbers correctly.

When students work on solving problems that are interesting to them, they are more likely to persist until they find a solution. One way to engage student interest is to look for tasks that have familiar contexts or that are connected to questions about which students are curious. Providing a real-world context helps students see mathematics as a useful tool rather than just something they "have to do for school." "Clowning Around," the problem on page 12, is an example of this type of problem. Additional problems with engaging contexts are available on the Web—see the *Resources* section for more information.

Understanding: Making mathematical thinking concrete

Mathematically proficient people understand mathematical concepts and ideas. They are able to make sense of and use the symbols, terms, and operations of mathematics. In school, students are introduced to many mathematical facts and procedures. By looking for connections and meaning rather than memorizing these concepts in isolation, students begin to build their understanding of mathematical ideas.

In the past, students frequently memorized facts or procedures *without* understanding. However, when students memorize rules they don't understand, their knowledge tends to be fragile (Bransford, Brown, & Cocking, 1999). They are not able to apply the facts or procedures they have memorized to unfamiliar problems or situations.

Students often arrive for homework help with assignments they don't understand and can't explain. If you are working with a small group, check with other students to see if they had a better or different understanding of the homework and/or what happened in school. If this strategy doesn't help, or if you are working one-on-one, the following approaches can help you find out more about what a student does and doesn't understand: initiating mathematical discussions and using models.

Initiating mathematics discussion

Talking about mathematics is one of the most effective ways for students to identify gaps in and deepen understanding. Explaining concepts in their own words helps students make sense of what they are learning. One way to begin a discussion is by asking students to explain how they got their answer.

Even a simple problem can be a jumping-off point for a discussion about mathematics. The following scenario shows how a student might respond to a prompt such as "How did you find the answer?" or "What is another way to solve the problem?" Students' explanations can be especially useful when they have an incorrect solution—as the example below illustrates.

Kelly solved the following multiplication problem like this:

$$\begin{array}{r} {}^{1}36 \\ \underline{\times\ 3} \\ 128 \end{array}$$

Can you tell why Kelly got the incorrect answer? It may not be obvious, so Aaron, the tutor, asks her to explain her thinking.

Kelly: *First I multiplied 6 x 3 and I got 18. I wrote down the 8 and carried the 1.*

$$\begin{array}{r} {}^{1}36 \\ \underline{\times\ 3} \\ 8 \end{array}$$

1 + 3 is 4 so then I multiplied 4 times 3 and got 12.

$$\begin{array}{r} {}^{\cancel{1}}\cancel{3}46 \\ \underline{\times\ 3} \\ 128 \end{array}$$

So the answer is 128.

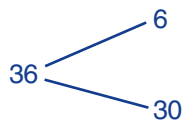
Aaron realizes that Kelly does not understand that the '1' she carried is actually one 10. She remembers something about adding it to the next number, but gets confused and adds before she multiplies. This is a common mistake. He decides to ask Kelly to solve the problem another way.

Aaron: *Now I would like you to check to see if your answer is correct. Can you try solving the same problem another way?*

Kelly hesitates. After waiting to verify that she is not sure what to do, Aaron tries a more specific prompt.

Aaron: *Why don't you try breaking 36 down into smaller numbers?*

Kelly: *I can break 36 down into 30 and 6*



Then I can work with those two numbers separately. 30 x 3 is the same as adding 30 three times, and the answer is 90. If I also add 6 three times I get 18. To make 18 easier to work with I can break it down into 10 and 8.

$$\begin{array}{r} 30 \\ 30 \\ \underline{+30} \\ 90 \end{array} \qquad \begin{array}{r} 6 \\ 6 \\ \underline{+6} \\ 18 \end{array}$$

10 8

I can add the 10 from the 18 to the 90 to get 100. Then I just add the 8 for the final answer: 108.

90 + 10 = 100
100 + 8 = 108

This example shows Kelly has an understanding of place value and can take apart or "decompose" numbers and put them back together with ease. Breaking numbers apart into 10s and ones is a powerful tool to help children understand how numbers work. In addition, this solution shows Kelly's knowledge that multiplication can also be understood as repeated addition.

To help Kelly understand her initial mistake, Aaron asks her to compare her two solution methods. They discover that when Kelly added the 1 from 18 to the 3 from 30 and then multiplied by 3, it was the same as changing the problem to 40+40+40 instead of 30 + 30 + 30. That is why she needed to multiply 30 times 3 first before adding the 1.

Using models

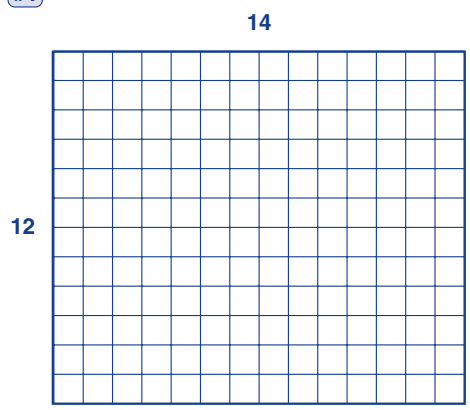
Encouraging students to draw a picture to represent a mathematical idea or to illustrate a problem can help them to develop better understanding of concepts. Sometimes this involves creating a mental or physical picture of what is happening in a mathematics problem. For example, it may be helpful for students learning about negative numbers to visualize a number line or to draw positive and negative charges.

Laura is learning how to multiply two-digit numbers in her mathematics class. However, she is having a hard time remembering all the steps in the formula. Henry helps her by demonstrating an array model (see illustrations for an example) for the problem 12x14.

Henry: *Using grid paper, you can show 12 x 14 by drawing a rectangle that has a length of 12 and a width of 14. Can you draw this rectangle?*

Laura: *Here it is.* (See diagram #1.)

#1



Henry: *How can we figure out the product of 12 x 14? How many grid squares are there altogether?*

Laura: *One, two, three, … 124, 125…*

(Students may be tempted to count by ones, but they soon realize that it's cumbersome, inefficient, and they may be prone to making mistakes.)

Henry: *Sometimes it's helpful to use numbers that are easier to work with, like 10s and ones. How many 10s and ones are there in 12?*

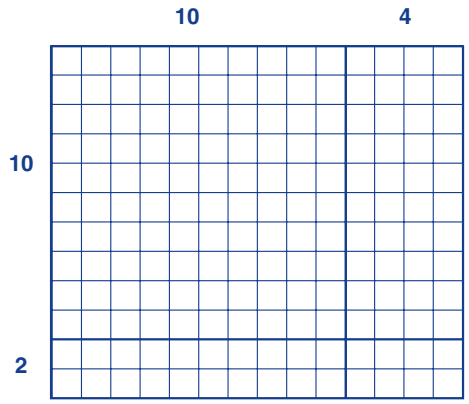
Laura: *One 10 and two ones*

Henry: *In 14?*

Laura: *One 10 and four ones*

Henry: *We can show this on the grid paper.* (See diagram #2.)

#2

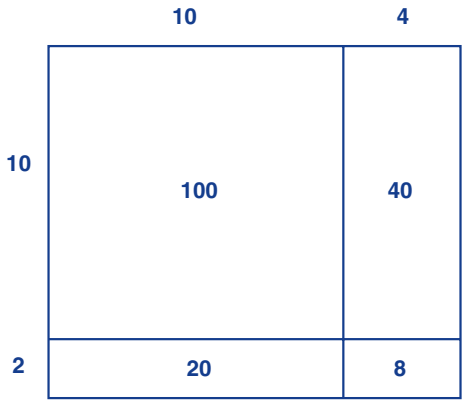


Henry: *By separating the 10s and ones, it is easier for us to figure out how many boxes there are altogether. What strategies can you use?*

Laura: *I can figure out that 10 x 10 = 100, then 4 x 10 = 40, then 2 x 10 = 20, and finally that 2 x 4= 8. Then I can add them together to get the answer.*

Henry: *Can you label each section and then show how you added them up?* (See diagram #3)

#3

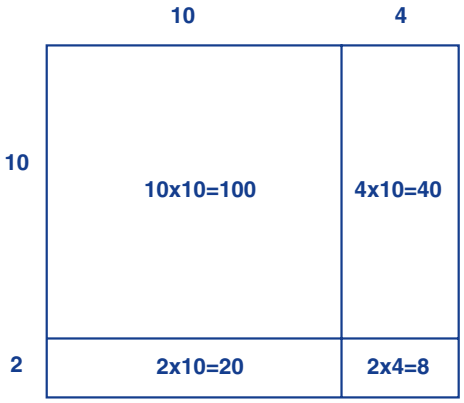


Henry: *In class, you may have seen someone use numbers to solve this problem. Where is each number in your array?*

$$\begin{array}{r} 12 \\ \times\ 14 \\ 48 \\ +\ 120 \\ \hline 168 \end{array}$$

Laura: *You get 8 when you multiply 2 by 4. You get 40 when you multiply 10 by 4. You get 20 when you multiply 2 by 10. You get 100 when you multiply 10 by 10. The 48 in the problem is when you add the 8 and 40. The 120 in the problem is when you add 100 and 20.* (See diagram 4)

#4



Henry: *That makes perfect sense to me!*

It is important for your students to make the connection between concrete models, like arrays, and more abstract representations, like numbers. Your goal is to build students' understanding of mathematical concepts like multiplication, and in doing so, support their use of more efficient strategies as their understanding becomes more secure.

Asking students to represent a problem with a drawing can help you identify breakdowns in understanding. For example, a student might have memorized 4 x 3 = 12, but be unable to represent that problem as four groups of three or three groups of four. Being aware of what a student *doesn't* understand can help you target your time together on areas he needs to improve.

Models can also help students demonstrate understanding, because they may be able to show what they know with a representation before they can explain it in words. For a fraction problem, students might demonstrate understanding by drawing a picture or by coming up with a story that illustrates the mathematics. Tutors can also use stories and pictures to help students make sense of a mathematics problem.

Margaret: *I don't understand how to do these division problems.*

Jill: *Let's try thinking of how division might help you to figure out a real situation.*

Margaret: *Okay. The first problem is 376 divided by 15.*

Jill: *Well, try thinking about this. Your school has 376 fifth-graders and 15 classrooms. You need to help the principal figure out how many students to put in each of those rooms. Maybe you can start by drawing the classrooms…*

Having manipulatives available for students to use is also helpful. Manipulatives help create a concrete representation of an abstract problem. This helps students connect the numbers and symbols they are using to the physical objects that the numbers and symbols might represent. See the box at left for more information about this strategy.

Manipulatives can be helpful in uncovering students' misunderstandings. When students use manipulatives to demonstrate a problem that they have solved incorrectly, they can uncover the problems in their own thinking.

For example, Will solved the following problem like this:

$$\begin{array}{r} 70 \\ \underline{-\ 54} \\ 24 \end{array}$$

Hope: *Can you tell me how you got your answer?*

Will: *Well, I know that when you have 4 and 0 it's always 4 and then 7-5 =2. So the answer is 24.*

Hope learns that Will is trying to apply the rule for adding zero to this subtraction problem.She decides to use some manipulatives to help him find his mistake.

What are Manipulatives?

Manipulatives are movable objects used by students to illustrate or act out a mathematical situation. Everyday objects like pennies, beans, toothpicks, or paperclips can be used as manipulatives. Some classrooms use commercially made manipulatives such as pattern blocks, linking cubes, or place value blocks. These items have particular shapes or sizes that are useful for showing mathematical concepts or relationships.

Why use manipulatives?

Manipulatives make mathematical concepts concrete for students and help them make sense of mathematical problems. When students are working on a new concept, the use of manipulatives enables them to build understanding or see connections to other concepts they already know.

How should I use manipulatives?

Have a few manipulative materials available for students to use if they choose to do so. If students are struggling with a problem, ask if they would like to try acting it out using manipulatives. Keep in mind that the purpose of using the manipulative is to help students build understanding. Resist the temptation to "show" students a particular way to use the materials. Instead, allow them to use them if and when they find them useful.

Hope: *Can you show me with this chart?*

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Will: *I start at 70 and I need to count back 54. I can count backward by five 10s for 50…60, 50, 40, 30, 20 and then 4 more and I end up at 16. So 16 is the answer. 70 – 54 = 16.*

Hope: *When you did the problem before, your answer was 24. Which answer do you think is correct?*

Will: *It looks like 16 is correct, but I followed the rule when I got 24.*

Hope: *Let's try it another way. Here are some dimes, nickels, and pennies. You have 70 cents to spend at the store and you are buying some candy and gum. The total is 54 cents. How much money will you have left?*

Will: *This is how much money I have when I go to the store.* He counts out 70 cents in dimes.

Now I am going to give you 54 cents for the soda and candy.

He counts out five dimes (10, 20, 30, 40, 50) and realizes he does not have 4 cents. He takes one of his dimes and trades it for 10 pennies. After the exchange, he recounts the dimes (10, 20, 30, 40, 50) and then four pennies.

Hope: *How much do you have left?*

Will then counts the remaining money, one dime and six pennies.

Will: *I would have 16 cents left. So I guess 16 is the right answer.*

Hope: *I'm going to write down what you just did, then we can compare it to the first time you solved the problem. I'm going to write 'd' for dimes and 'p' for pennies.* Hope writes:

7d + 0p (exchange 1d for 10 p)	→	6d + 10p
– (5d + 4p)	→	– (5d + 4p)
		1d + 6p

Hope: *So 1 dime plus 6 pennies is 10 cents plus 6 cents and you have 16 cents.*

Will: *I see it now. When I did the problem the first time I didn't exchange the zero for 10 ones.*

Hope: *Great job! You worked really hard to find out the answer. Let's try another problem like that one.*

Computing: Building mathematical fluency

Mathematically proficient people have the ability to compute flexibly, accurately, efficiently, and appropriately. In order to have fluency with computation, students need to perform efficiently, which comes from a combination of the ability to reason numerically and having memorized some facts and procedures (NCTM, 2000).

However, the ability to speedily work through a sheet of problems or to recite the multiplication tables from memory is not sufficient. Students need to be able to perform calculations and other procedures accurately, but they also need to know when and how to use them. Computing and understanding go hand in hand.

Volunteers can help students strengthen this strand of mathematical proficiency by creating mini-lessons or activities that isolate and practice the specific skills with which students are struggling. To create an effective mini-lesson:

- Look at the assignment the student is struggling with and isolate the specific skill or strategy.
- Decide on an alternative activity that is unrelated to the assignment at hand — this might be a simpler problem using the same skill or a game that utilizes the skill.

- Ask the student how she figured out the answer and discuss the strategies she used.
- Ask the student to apply this new knowledge to the original assignment.

Reframing challenging activities in a mini-lesson or game allows students to practice the same skills in a less stressful context before applying what they learn to their homework or schoolwork.

Providing varied forms of practice

When students memorize procedures before they understand the mathematical ideas behind them, they have more difficulty developing understanding (Kilpatrick, Swafford, & Findell, 2001). Following are some techniques for providing students with opportunities to practice computing skills.

- Use dominoes or dice to generate problems. Roll two or more dice or turn over a domino and have the student find the sum. Play a "running total" game. Keep rolling dice (or turning over dominoes) and adding the new sum to the previous total until you reach a target like 50 or 100.
- Use playing cards to generate number problems. Remove the face cards and let the ace represent 1. For practice with basic number combinations, place two cards face up and ask the child to add, subtract, or multiply them. For practice with multi-digit computation build a two-digit number using two cards. Build another two-digit number using two more cards. Now have the child add or subtract these numbers. For practice building flexibility with numbers, place four cards face up and turn over one more card to be the target number. Challenge students to use all four numbers and any operations to produce the target number.
- Use a problem, such as "How many cats and dogs?" There are 12 pets—some of them are cats and some of them are dogs. Ask students: "How many cats and dogs could there be?" The students then find all the number combinations that add up to 12.
- Mathematical games are another way to provide students with varied forms of practice. One of the benefits of using games is that they are fun and non-threatening, and therefore create a positive environment that enhances learning. Games also contribute to students' positive attitudes toward mathematics. Following are two sample games that can help students understand addition:

FINISH THE 10

Materials: Playing cards (let A stand for 1 and Q stand for 0, remove J, K) Counters (beans, pennies, etc).

Players: 2–4

Directions:

- Player 1 draws a card and claims the number of counters that "finish the 10" with the value drawn. For example: If Player 1 draws a 6, he/she collects 4 counters because 6 + 4 = 10. (If necessary, write out 6 + [ ] = 10, to help the student understand the goal of the game.)
- Other players, in turn, do the same.
- Players keep track of how many counters they have and play continues until one player collects 30 counters.

Questions:

- What strategies did you use to figure out how many chips you needed to take?
- How did you keep track of how many counters you had?
- How do you know how many more counters you need?

Winner: The first player to collect 30 counters.

GO FISH FOR 10

Materials: Playing cards (let A stand for 1 and Q stand for 0, remove J, K)

Players: 2–4

Directions:

- Deal seven cards to each player and form a draw pile with the rest of the deck. Each player looks for two-card combinations that sum to 10 and lays these down in front of him/her.
- Play as for a normal game of "Go Fish" except each player must ask for the card that completes a two-card combination that makes 10. For example, Nina might ask Roberto, "Do you have a 7?" because Nina has a 3. If Roberto has a 7, he gives it to Nina. If not, Nina draws one card from the draw pile.
- Play continues until the draw pile is exhausted. (If a player runs out of cards, he draws one from the draw pile in order to stay in the game.)

Questions:

- What patterns do you notice?
- What are the combinations that make 10?

Winner: Player with the most pairs of 10 combinations after all cards have been played.

Applying: Using mathematics to solve real-world problems

Mathematically proficient people are able to use what they know about mathematics to solve problems. Problem solving can be defined as engaging in a task without knowing in advance what method to use (NCTM, 2000). It is the type of mathematics activity that people confront in their jobs and their daily lives.

National service members and volunteers can help students strengthen this strand of mathematical proficiency by providing complex problems and teaching problem solving strategies.

Providing complex problems

Problem solving requires students to think about and apply mathematics. The context of a problem helps students make meaning of what they are learning and piques their interest in solving the problem.

In order to become proficient with solving problems and applying the mathematics they learn, students need many opportunities to work on complex problems. The problem below is an example of a problem-solving activity. Many other tasks for all grade levels are available on the Web—see the *Resources* section for more information.

Clowning Around

Here's something to think about…

Your school is putting on a parade and your class is in charge of deciding how many clowns will be in the parade and what they will ride on. Clowns can ride on unicycles, bicycles, or tricycles. The Spinning Wheels Bike Shop has donated 15 wheels to your school to build the cycles.

- *How many unicycles, bicycles, and tricycles can be built for 8 clowns, using only 15 wheels?*
- *Find as many possible combinations of unicycles, bicycles, and tricycles as you can for 8 clowns with the 15 wheels.*
- *Show all your work and tell how you solved the problem using numbers, pictures, and words.*

If you think you have found all the possible combinations of unicycles, bicycles, and tricycles, tell why you think there are no other combinations that will work.

Volunteer tutors and mentors may be unsure how to help students work on problems such as "Clowning Around." A first step is helping students get started with a complex problem. Try the following steps:

- Have the student read the problem out loud, or read it together.
- Ask the student to explain the problem in her own words.

- Brainstorm ways to get started.
- Come up with a plan for solving the problem.

Teaching problem-solving strategies
To become proficient in applying mathematics, students need to know a variety of strategies for solving problems, as well as when to use them. An essential aspect of mathematics education is teaching students how to use strategies such as the following:

- Using diagrams
- Acting out the problem using manipulatives
- Looking for patterns
- Listing all possible answers
- Working backward
- Guessing and checking
- Creating an equivalent problem
- Creating a simpler problem

Tutors can help students by discussing and drawing attention to strategies throughout the problem-solving process. These are some questions that you might want to ask at the beginning, middle, and end of an activity.

Before students get started:
What tools might help you to solve this problem?
What other problems have you solved that are similar to this one? How did you figure them out?

While students are working:
What might help you find out what you need to know?
Can you explain the strategy you are using?
What else could you try?
What would happen if…?

After work is complete:
How did you get your answer?
Can you show me that it makes sense?
What strategies did you use?
What other strategies could you have used?

Reasoning: Demonstrating mathematical knowledge

Mathematically proficient people are able to explain their thinking and justify their solutions. They are also able to use reasoning to figure out unfamiliar situations by drawing on their existing knowledge. Reasoning is the link that connects mathematics into a coherent body of knowledge. It links the facts, procedures, concepts, and solution methods (Kilpatrick & Swafford, 2002).

Tutors can help students strengthen this strand of mathematical proficiency by expecting justification for solutions.

Expecting justification
Students develop reasoning abilities by explaining their thinking and their solutions to others, rather than just doing practice problems. Students also learn how to justify their conclusions. To hone this habit, students should provide their rationale as part of every answer.

One way to help students demonstrate their reasoning abilities is to have them talk about the steps they are taking as they solve a problem. You might also ask students to create a representation, such as a picture, to show how they came up with their solution or how they know that their answer is correct (Kilpatrick, Swafford, & Findell, 2001). This can be especially helpful for younger students.

Students' ability to evaluate their own or someone else's reasoning is also important. One way to do this is to have students compare their solutions or their methods with other students. You might ask: "How are these two solutions alike? How are they different?" If you work with students one-on-one, you can provide problems in which the student analyzes someone's thinking—the following is an example.

Which is larger: 1/2 or 1/3?

Holly thinks that 1/3 is larger because 3 is larger than 2.

Diego thinks that 1/2 is larger because a piece that is half of a whole is larger than a piece that is one third of a whole?

Who is correct? How do you know?

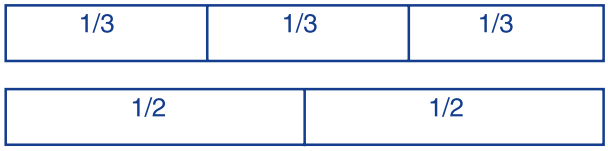
Can you show Holly and Diego the correct answer with a picture?

Talking about 1/2s and 1/3s can be very confusing, especially when the student does not know the whole.

There are several ways to help students visualize this problem. One way is to make fraction strips.

Give the student several strips of paper of equal lengths. Explain that the entire length will represent one. Ask the student to fold the strip in such a way to represent half and label the parts. It should look like this:

Ask the student to fold the paper in such a way to represent 1/3 of the strip. It should look like this. Note: It is sometimes very difficult to get 1/3s. The student will more than likely have to do this using trial and error.



Now compare the two strips. Which is larger, 1/2 or 1/3? The important thing to remember is that the whole must always be the same when comparing fractions.

Students who are able to justify their reasoning can talk about the mathematical ideas in the task and the procedures they use, provide a good explanation for what they are doing, and justify the claims in their solution.

You can help students learn how to justify their work by framing your conversation with appropriate prompts and questions. To draw out this student's reasoning on the "Clowing Around" problem, you might ask:

- *Can you tell me more about the patterns you see?*
- *How did you get your first answer?*
- *Did you always start with the tricycle?*
- *How do you know there can't be any more? How would you prove it?*
- *Can you show me how you started to think about this problem?*
- *Do you think there has to be at least one of each? Why?*
- *What did this problem show you about patterns?*

Conclusion

Most national service members and volunteers encounter mathematics as part of homework help or other academic support in out-of-school time. To a large extent, the content you focus on will be determined by the assignments your students are working on in class. When you encounter a confusing assignment, need a new approach to a persistent challenge, or simply want to add some variety into your homework help routine, the strategies in this article can help. While the skills and examples included are most appropriate for elementary school students, the broad strands of mathematics instruction presented here—engaging, understanding, computing, applying, and reasoning—and the concrete strategies that support each strand, can be applied to most any assignment or context. Students need a strong grounding in mathematics to succeed in a range of real-world settings. With your help, they'll be ready!

Resources

Afterschool Tutoring Toolkit: Math
www.sedl.org/afterschool/toolkits/math
Created by McREL for the National Partnership for Quality Afterschool Learning, this Web site contains promising practices, sample lessons, and tools for supporting math in out-of-school time.

Ask Dr. Math
http://mathforum.org/dr.math
At this site, visitors can post a question about mathematics or browse through the answers to previous posts. This program is part of the Math Forum Web site, which provides additional mathematics education resources.

Balanced Assessments
http://balancedassessment.concord.org
This Web site has a variety of problem-solving tasks for all grade levels.

Cool Math
www.coolmath.com
This site features puzzles, games, and fun mathematics lessons.

Figure This!
www.figurethis.org
This site features interesting math challenges that are designed for families of middle-school students. The site also features suggestions for helping with math homework and examples of math in literature.

Fun Mathematics Lessons
http://math.rice.edu/~lanius/Lessons
This large collection of problem-solving tasks also includes a few with Spanish versions.

Great Sites for Math Teachers
people.clarityconnect.com/webpages/terri/sites.html
A teacher maintains this collection of useful Web sites, organized by category. The site is intended for the middle school level (grades 6–8) but many of the links are appropriate for all ages.

Hands-on Math
www.dpgraph.com/janine/mathpage/handson.html
These mathematics problems were developed by a teacher for elementary level students. They include tasks on geometry, number patterns, and topology.

Helping Children Learn Mathematics
darwin.nap.edu/books/0309084318/html/
For more detailed information about the five strands of mathematical proficiency, this resource from the National Research Council is available online.

Math Partners
www2.edc.org/mathpartners
This mentoring curriculum includes diagnostic assessments and tasks for grades K–8, as well as tips and suggestions for mentors.

NCTM Illuminations
http://illuminations.nctm.org
The Illuminations Web site is maintained by the National Council of Teachers of Mathematics. Visitors will find lessons and online activities for students in grades preK–12.

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